Section V.H

Expected Sensitivity and Systematic Effects

1.A Scintillation Signal Sensitivity

Scintillation Rate

The system frequency measurement sensitivity can be estimated using the mathematical techniques outline in [1]. At the start of the measurement, we assume N UCN are trapped in each storage vessel, and scintillation light is produced as previously described through ^{3}He-UCN spin-dependent reactions. We also include here scintillation light produced by UCN beta decay, and possible background scintillation due to gamma Compton scattering and apparatus beta-activation. This treatment is slightly different from that presented in [2] because there we assumed it would be possible to discriminate between the ^{3}He-UCN reactions from UCN and neutron activation beta decay, and from Compton backgrounds; as we now expect 5-10 photoelectrons per ^{3}He-UCN capture event, the possibility of such discrimination remains an open question. Furthermore, the analysis presented here is for free-precession of the ^{3}He and UCN (dressing as described in [2] is not used; SQUID magnetometers will pick up the ^{3}He magnetization signal and therefore provide a direct measurement of the average magnetic field seen by the ^{3}He which, to high accuracy, is the same for the UCN as described later in this Section).

The net scintillation rate from each cell can be written

$$\Phi = \Phi_B + Ne^{-\Gamma_{ave}t}\left[\frac{1}{\tau_\beta} + \frac{1}{\tau_3}[1 - P_3 P_n e^{-\Gamma_p t}\cos(\omega_r t + \phi)]\right] \tag{V.H.1}$$

where Φ_B is the background rate (assumed constant), Γ_{ave} is the total of the UCN β decay rate, cell loss rate (accounting for wall losses and upscattering from superfluid ^{4}He), and average ^{3}He absorption rate

$$\Gamma_{ave} \equiv \frac{1}{\tau_\beta} + \frac{1}{\tau_{cell}} + \frac{1}{\tau_3}$$

and $\tau_\beta \approx 885$ s, $1/\tau_3 = 2.4 \times 10^7\ X$ where X is the ^{3}He concentration, P_3 and P_n are the ^{3}He and UCN polarizations, Γ_p is the sum of the ^{3}He and UCN polarization loss rates, ω_r is the difference in precession frequencies, and ϕ is an arbitrary phase. The variance in the determination of ω_r together with the high voltage magnitude determines the experimental sensitivity. At this point, it is useful to define

$$\Gamma \equiv \frac{1}{\tau_\beta} + \frac{1}{\tau_{cell}}.$$

We can estimate the experimental sensitivity simply by making some reasonable assumptions. A real experiment will be slightly more sensitive by careful choice of cell filling time, total measurement time, and ^{3}He concentration. But for our purposes here, we assume that UCN are accumulated for two UCN storage lifetimes ($2/\Gamma$). We adjust X

so that $\tau_3 = 1/\Gamma$. The measurement time is taken as $T_m = 1/\Gamma$. We also assume that $\Gamma >> \Gamma_p$ so spin relaxation can be ignored in the following discussion.

Sample Size and Polarization

Another change from the sensitivity calculation in [2] is required because we now plan to produce polarized UCN from polarized 8.9Å neutrons. The initial UCN polarization and density, assuming UCN are accumulated with polarized ^{3}He in the superfluid bath, can be determined by considering the time dependence of the number of UCN with spin parallel (N_+) and antiparallel (N_-) to the ^{3}He initial polarization:

$$\dot{N}_+ = \frac{(1+P_{cn})PV}{2} - \left[\Gamma + \frac{1-P_3}{\tau_3}\right] N_+ = P_+V - \Gamma_+ N_+$$

$$\dot{N}_- = \frac{(1-P_{cn})PV}{2} - \left[\Gamma + \frac{1+P_3}{\tau_3}\right] N_- = P_-V - \Gamma_- N_-$$

where P_{cn} is the cold neutron beam polarization, P is the total UCN production rate (UCN/cc sec), and V is the storage chamber volume. These equations are easily solved, yielding

$$N_+(t) = \frac{P_+V}{\Gamma_+}(1 - e^{-\Gamma_+ t})$$

$$N_-(t) = \frac{P_-V}{\Gamma_-}(1 - e^{-\Gamma_- t}).$$

In the limit $P_{cn}, P_3 \approx 1$,

$$N(t) = N_+(t) + N_-(t) \approx N_+(t) = \frac{PV}{\Gamma}(1 - e^{-\Gamma t}).$$

In this approximatation,

$$P_n(t) = \frac{1 - N_-(t)/N_+(t)}{1 + N_-(t)/N_+(t)} \approx 1 - 2\frac{N_-(t)}{N_+(t)}$$

$$= 1 - 2\left[\frac{1-P_{cn}}{1+P_{cn}}\right]\left[\frac{\Gamma + (1-P_3)/\tau_3)}{\Gamma + (1+P_3)/\tau_3}\right]\left[\frac{1-e^{-\Gamma_- t}}{1-e^{-\Gamma_+ t}}\right]$$

which, when $\tau_3 = 1/\Gamma$ and $t = 2/\Gamma$, implies for the initial number of UCN and polarization

$$N = 0.86PV/\Gamma; \qquad P_n = 1 - (0.4)(1 - P_{cn})$$

which, if $P_{cn} = .90$, $P_n = 0.96$.

The oscillating signal described by Eq. (V.H.1) commences when $\pi/2$ pulses are applied, flipping the spins perpendicular to the applied static field.

Frequency Variance

By use of the formalism presented in [1], the frequency variance per measurement cycle of lenght T_m can be estimated. This requires the average oscillating signal amplitude $A(t)$, and the constant (or slowly) decaying background signal $\overline{I(t)}$, which we will assume is principally due to beta decay, imperfect polarization, and the constant (non-oscillating) component in Eq. (V.H.1).

From Eq. (1), the signal amplitude due to ^{3}He absorption (one-half peak-to-valley scintillation difference between spins parallel and antiparallel) is (assuming $\Gamma - p = 0$)

$$A(t) = \frac{NP_3P_n}{\tau_3} e^{-\Gamma_{ave} t}$$

while the average per cycle scintillation rate, due to beta decay and ^{3}He absorption, is

$$\overline{I(t)} = \Phi_B + Ne^{-\Gamma_{ave} t}(1/\tau_\beta + 1/\tau_3)$$

Over the measurement time T_m, the average amplitude and average current is, recalling that we assume $\Gamma = 1/\tau_3$

$$\bar{A} = (0.43)\frac{NP_3P_n}{\tau_3}$$

and

$$\bar{I} = \Phi_B + 0.43N\left[\tau_\beta^{-1} + \tau_3^{-1}\right].$$

From Eq. (9) of [1], the average uncertainty in the frequency after observing the oscillating scintillation for T_m seconds is, recalling that $\tau_3 = 1/\Gamma = T_m$,

$$(\Delta f)^2 = \frac{6}{\pi^2}\left(\frac{\bar{I}}{\bar{A}^2}\right)\frac{1}{T_m^3}$$

$$= \frac{1.41}{P_3^2 P_n^2 N T_m}\left(\left[\frac{\Phi_B}{0.43N(T_m^{-1} + \tau_\beta^{-1})} + 1\right](1/\tau_\beta + 1/T_m)\right) \text{ Hz}^2 \qquad \text{(V.H.2).}$$

Taking $T_m = \tau_3 = 1/\Gamma = 500$ sec, $P = 1$/cc/sec, $V = 4,000$ cc and assuming $\Phi_B/N << 1/\tau_\beta$, then

$$\Delta f = 2.6\ \mu\text{Hz}$$

which, assuming 50 kV/cm, corresponds to 2σ EDM sensitivity of 10^{-25} $e$cm, for each cell. Now, we are comparing two cells with oppositely directed electric fields, so the sensitivity per measurement cycle is

$$7 \times 10^{-26}\ e\ \text{cm}.$$

Each measurement cycle requires 1,500 sec, so the experimental sensitivity of the measurement is

$$9 \times 10^{-27}\ e\ \text{cm} \cdot \sqrt{\frac{\text{day}}{T}} \qquad \text{(V.H.3)}$$

or, in $T = 100$ days of running, the (2σ) limit will be 9×10^{-28} e cm, a factor of at least 50 improvement over the present experimental limit. The calculation presented here is not optimized for accumulation vs. measurement time, and not optimized for ^{3}He density; on the other hand, certain idealizations (e.g., polarization does not decay, activation background is small) were assumed. However, this simplified calculation clearly shows the expected experimental sensitivity. It is a worst-case estimate in the sense that we assumed the beta-decay background can not be discriminated from the ^{3}He-UCN reactions.

Operating the experiment at a more intense neutron source will improve the sensitivity; the purpose of this calculation was to show the possibilites at LANSCE. The cold neutron flux available at the Institut Laue-Langevin is approximately 50 times greater, corresponding to a factor of 7 increase in sensitivity. In addition use of the dress-spin technique described in [2] offers a significant increase in sensitivity.

1.B Required SQUID Magnetometer Sensitivity

It is necessary to use the ^{3}He precession signal to determine the average magnetic field affecting the UCN precession. Ideally, the field would be constant, however, there are changes due to finite shielding of ambient magnetic field changes, and systematic fields due to leakage currents associated with the application of high voltage.

Because the ^{3}He and neutron magnetic moments are equal to within 10%, and the electric field does not affect the ^{3}He precession, we need only know the difference in magnetic Larmor precession frequencies to high accuracy. The sensitivity per measurement cycle is 2.6 μHz, so the minimum required accuracy on δB is

$$\delta B = \frac{2.6\mu\text{Hz}}{|\gamma_3 - \gamma_n|} = 8\text{nG}$$

per measurement cycle; practically, we would like the accuracy on δB to be a factor of three smaller so that it does not contribute noise to the measurements.

Our plan is to use SQUID magnetometers to pick up the ^{3}He precession. The uncertainty δB is related to the uncertainty on the determination of the ^{3}He precession frequency,

$$\delta f_3 = \gamma_3 \delta B = 2.6 \times 10^{-5}\text{Hz}$$

which, as expected, about a factor of ten larger than the neutron frequency uncertainty.

The amplitude of the ^{3}He magnetization signal can be estimated as follows. The magnetization of the sample is

$$M = \frac{h\gamma_3\rho_3}{2} = \frac{h\gamma_3}{2}\frac{9.2 \times 10^{14}(\text{ s/cc})}{\tau_3} = 2.3 \times 10^{-11} \text{ cgs units.}$$

It is reasonable to expect that the magnetic induction at a region close to the cell, coupled to the SQUID pickup coil, is something like 1/2 the value at the surface of the cell, so

$$B_P = 4\pi M/4 = 7.2 \times 10^{-11}\text{G}.$$

If the pickup loop has inductance L_p and the SQUID input inductance is L_i, the flux coupled to the SQUID is

$$\Phi_S = \frac{MB_P A_p}{L_p + L_i}$$

where M is the SQUID mutual inductance. Typically, $M = 10$ nH, $L_i + L_p \approx 500nH$. This implies, in terms of 10^{-6} of a flux quantum $\mu\Phi_0 = 2.07 \times 10^{-11}$G cm^2, a pickup loop area (assuming $L_p < L_i$ so we can ignore the change in $L_p + L_i$)

$$\Phi_S = 7.2 A_p \ \mu\Phi_0,$$

with A_p in cm^2, which is the signal to be expected at the SQUID. We can use Eq. (11) of [1] to determine the maximum noise per root Hz bandwith required to measure the ^{3}He precession frequency to 26 μHz accuracy in a measurement time $T_m = 500$ s,

$$(\delta f_3)^2 = (26\ \mu\text{Hz})^2 \left(\frac{n_1}{A}\right)^2 \frac{3}{\pi^2}\frac{1}{T_m^3} = \left(\frac{n_1}{(7.2\mu\Phi_0 A_p)}\right)^2 \frac{3}{\pi^2}\frac{1}{(500\ \text{s})^3} \tag{V.H.4}$$

$$\rightarrow n_1/A_p = 4\mu\Phi_0/\sqrt{\text{Hz}}/\text{cm}^2$$

and taking 1/3 this value to ensure no extra noise is introduced implies

$$n_1/A_p = 1.3\mu\Phi_0/\sqrt{\text{Hz}}/\text{cm}^2. \tag{V.H.5}$$

A Conductus 1020 has $n_1 = 3\mu\Phi_0/\sqrt{\text{Hz}}$ implying a 2 cm^2 pickup loop is required to attain the requisite signal-to-noise, for which $L_p = 30$ nH.

This calculation assumes that the ^{3}He polarization does not decay significantly over the measurement period.

2. Systematic Effects

In this section, we will address possible systematics that are "fundamental" as opposed to the usual concerns of external magnetic fields associated with reversing high voltage apparatus, etc. Of course, such effects can be important, but for the proposed experiment, will be largely suppressed by the internal ^{3}He comagnetometer. Here will will address issues that lead to real differences in the effective magnetic field seen by the ^{3}He and UCN.

2.A. Pseudomagnetic Field

The effective UCN potential is given in terms of the coherent scattering length as

$$U_f = \frac{2\pi\hbar^2}{m}\rho < a >$$

where m is the neutron mass, ρ is the density and $< a >$ is the average coherent scattering length. Each spin state of every consituent contributes to this potential; in the case where a constituent is polarized, there will be a different potential for each spin state; this energy difference creates a pseudomagnetic field. For polarized ^{3}He,

$$a_+ = 3.0 \pm 0.1 \text{ fm}$$

$$a_- = 8.2 \pm 0.3 \text{ fm}$$

which leads to a pseudomagnetic spin precession frequency of

$$6.6 \times 10^6 X \text{Hz}$$

where $X \approx 10^{-10}$ is the ^{3}He concentration. Thus there is a shift in frequency of 660 μHz when the ^{3}He polarization lies along the static magnetic field. Note that this frequency shift is not dependent on the electric field, but can introduce noise into the system if it varies between the two cells between fillings.

This rather large value is suppressed by a number of factors. First, the ^{3}He spins are flipped into the plane for the free precession measurement, so the average field seen by the UCN has near-zero average; the only contribution to the precession frequency is that due to the imprecision of the $\pi/2$ pulse. Achieving 5% accuracy for this pulse reduces the pseudomagntic precession to 33 μHz. Furthermore, the cells will be filled with nearly exactly the same ^{3}He density because both are filled from the same source, so the difference frequency is even smaller; we can assume the initial ^{3}He density is the same within 5%. Furthermore, the relative difference in the $\pi/2$ pulse between cells can be accurate to within 1%. These various factors bring the pseudomagnetic precession frequency difference to less than 1 μHz, which is sufficiently below the expected accuracy of about 2 μHz per measurement cycle so that no extra noise will be introduced. However, the importance of comparing two cells filled from the same source is evident.

It should be noted that EDM experiments in the presence of a large pseudomagnetic field are not without precedence. The ^{129}Xe EDM experiment [3] suffered a spin shift of about 5 mHz while the final experimental sensitivity was around 2×10^{-26} $e$cm with a field of 2 kV/cm. This implies that the spin shift did not contribute noise at the level of 0.04 μHz.

The direct ^{3}He magnetic field and its effect on the UCN is too small to be of concern compared to the pseudomagnetic field.

2.B. Gravitational Offset and other Spatial UCN/^{3}He Differences

Because the UCN kinetic energy is so low, the density "sags" under the influence of the Earth's gravitational field. The ^{3}He energy (effective temperature) will be around 0.3 K compared to 3 mK for the UCN, so the effect of gravity on the ^{3}He distribution is extremely small. The shift in center of mass of UCN in a storage chamber of height h was estimated in [4], p. 93:

$$\Delta h = \frac{mgh^2}{3kT}$$

so for $h = 10$ cm, 3 mK, implies $\Delta h = 0.13$ cm. The principal concern is that, if there is a spurious field from leakage currents, the ^{3}He and UCN will not average it in the same way.

The problem cannot be solved exactly because we don't know what the leakage current distribution will be, but we can estimate the effect.

The usual rule of thumb is that the current is assumed to flow in a 1/4 turn loop around the cell; assuming a current of 1 nA which is likely achievable at low temperature

$$B_{\rm sys} = \frac{1}{4}\frac{2\pi I(3\times 10^9 \text{ statamp/amp})}{cR} \approx 31\text{pG}$$

This corresponds to an EDM of about 10^{-27} e cm at 50 kV/cm. Now we must factor in the spatial difference to get the net systematic shift between the ^{3}He magnetometer and the UCN. The shift could be first order in the displacement, or (0.13 cm/10 cm), up to factors of order unity, leaving a maximum residual systematic error of 10^{-29} e cm, which is comfortably below the anticipated statistical limit.

Our efforts to detect a non-uniformity in the ^{3}He distribution in a test cell (by use of neutron tomography) showed no substantial effects. In fact, it is anticipated that the ^{3}He will be repelled by the superfluid-cell interface. Unlike the case of most atoms that experience a Van der Waals attraction, or even near chemical binding, to walls of storage cells, the case of ^{3}He in superfluid helium is special.

Another concern is the effects of unavoidable small heat currents on the ^{3}He distribution. At our anticipated operating temperature of less that 0.3 K, these effects are extremely small, with the effect of heat decreasing as the temperature T^{-7}.

2.C. $v \times E$ effects

The final systematic we will consider involves the motional or $v\times E$ magnetic field. The problem is that the UCN and ^{3}He atoms have very different average velocities. However, the motional field is randomly fluctuating because the velocity changes on subsequent collisions with the cell walls. The problem is discussed in [3], Sec. 3.5.3, and in [5], where it is shown that the effect is "quadratic" in that it is proportional to the square of the electric field. The direct motional field adds in quadrature with the static field and is reduced by a factor $x^2 = (\omega\tau_c)^2$ where ω is the Larmor frequency and τ_c is the time between subsequent wall collisions; this relation holds when $x < 1$. In [3] and [5] it is shown, extrapolated to a 50 kV/cm field, that the electric field must reverse with an accuracy of 10% to maintain a systematic shift below 5×10^{-28} $e$cm. This is a modest requirement on the electric field reversal accuracy.

Another point worth mentioning is that the relative shift will be temperature dependent when the temperature is high enough so that the ^{3}He atoms move diffusively in the superfluid bath. In this limit, the time between collisions that substantially change the ^{3}He velocity, hence τ_c, varies rapidly with temperature. In fact it might be possible to "tune" the temperature to a value where the relative shift is the same for the ^{3}He atoms and the UCN.

References

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